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Applications of Queuing Theory and Discrete Event Simulation in Health Care Units of Pakistan

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***Abstract*-** In the present research, the contribution of queuing theory and discrete event simulation in the improvement of healthcare was discussed in detail in the light of previously conducted research. This narrative literature review was conducted on the available research on the problems of healthcare in Pakistan and the analysis of evidences which are obtained from queuing theory and discrete event simulation (DES) research. Data was collected from the reports of world health organization and World Bank group and it was organized into tables by using MS excel. Mismanagement of the resources and the queuing system was highlighted to be the main reason for low quality of the healthcare service delivery in public sector hospitals of Pakistan. Behavior of staff with the arriving patients was reported to be irritating in public sector. Moreover, delayed service, long waiting times and less departmental capacity (at emergency, OPDs and laboratories) are the problems faced by the patients. On the same time, number of doctors is also less than required. When the citizens come to the healthcare facilities, at the point of service or distribution, overcrowding scenario is observed to an increasing extent. Patients are delayed at the public healthcare units long before they are served by medical staff. The delay is the main issue and that can be obviously understood that it is due to poor design or mismanagement of queuing system. For all these problems, queuing theory is the best tool but nowadays, simulation methodology has taken over the side of queuing theory very precisely. In this regard, this review contributes in highlighting the problems of healthcare in Pakistan and so it focus on the solution provided by the previous researchers as well. This review paper presents the overall picture of healthcare delivery system of Pakistan. Since, the issues of intensive care units (ICUs) and emergency departments (EDs) at the hospitals have specific issues of their own. The major limitation of this research paper is that it doesn`t present the in-depth understanding of problems of each facility. When the system (healthcare facility) is congested, patients wait more in the queues and system in order to get served. AT ICUs and EDs patients are in critical conditions and if the patients are made to wait in that condition, anything can happen. In this regard, it is suggested that review of problems of ED and ICUs should also be reviewed specifically so that the problems can highlighted for the greater good of nation. After the deep review of literature, it was clear that none of the literature reviews discussed the healthcare problems in the respective country and the review of contribution of related methodology at the same time. The contribution of present research in highlighting the problems of healthcare delivery system in Pakistan and review of queuing theory and queuing simulation cannot be ignored.

***Keywords*-** *Queue, Simulation, Waiting Time, Healthcare, Hospitals*

I. INTRODUCTION

World health Organization (WHO) has defined healthcare in its report of 2000 that “all organizations, institutions and resources that are devoted to producing health actions” (Musgrove et. al., 2000). Health is termed as the functional fitness that stress on personal and social resources, as also physical capabilities. In the race of human beings, the degree of physical, emotional, mental and social ability of and individual to deal with his/her surrounding environment is known as health (Naz et. al. ,2012). Health is defined by World Health Organization (2010) as “a state of complete physical, mental, and social well-being, and not merely the absence of diseases or injury” (WHO, 2010). Good health is a requirement for respective working of any individual or society, if the health of individuals is good then they can engage themselves in various types of activities (Naz et. al. ,2012). But if they are injured, distressed or ill, they may face a certain limitation in their lives and in comparison to health other pursuits in one`s life are meaningless (Jalal et. al., 2009). Healthier people can only be the source for making the healthier society, therefore, the Government should frame the policy and regulation on patient care and also take step of developing mechanism to insure compliance (Nasim & Janjua, 2014). Access to healthcare services is dependent on accessibility of the service for example, availability of doctors, healthcare centers and also hospitals. Research has concluded that due to lack of interest of

staff and unavailability of facilities are the main reasons of dissatisfaction of patients in Pakistani public healthcare system (Ahmad et. al. , 2013). It is always expected from the health care systems to serve the medical needs of the population effectively and efficiently. One of the major objective of health care systems is to improve the health of the living people of the country (Musgrove et. al., 2000), (Mashhadi et. al., 2016).

Good health is necessary for the citizens of any country; if the people are healthy then they can contribute in the improvement of national economy. Healthcare system is the major sector which plays an important role in providing and maintaining good health of citizens of country. There are two healthcare sectors exist in Pakistan i.e. public and private. Long before, private hospitals were in a little quantity but due to the poor healthcare services, it attracted the people by providing better healthcare services to them. These two healthcare sectors were compared and it was concluded that it will be wrong to say that the private sector is more efficient and effective in terms of the available facilities and infrastructure; poor healthcare services in the public sector hospitals are due to the mismanagement of the medical personnel, which leads to patient dissatisfaction and they cannot have the desired level of service.

Pakistani public health care system facilitated with the various facilities and resources which are certainly not enough and the available facilities and resources are at the point of mismanagement. This is the reason patients arriving at public sector hospitals face a lot of problems. Delay is one of the major problems, some of the patients die waiting for their turn of service. Service is delayed when the service demand is more than the available capacity. When there is low capacity in comparison to the demand then the queue will form in the system. For the analysis and study of queues, queuing theory is used. Queuing theory was formulated by Danish Engineer A.K Erlang in 1913. It is the mathematical tool to solve the problems of queuing systems. The optimum solutions are figured out by the help of queuing theory in the form of performance measures. Unlike queuing theory, simulation is also justified approach to detect the bottlenecks in the queuing system or the process of service as reported by many authors.

II. AIMS AND OBJECTIVES

Since healthcare system problems are main problems of any country because without healthier citizens any country will lag behind. In this regard, aim of this research was to highlight the problems of private and public healthcare in Pakistan and at the same time, solutions of various problems were also discussed in the light previously conducted research.

- To identify the problems of Pakistani private and public healthcare delivery systems
- To review the applications of queuing theory and discrete event simulation in solving the problems of healthcare delivery systems across the globe
- To suggest the space for the improvement in healthcare delivery system

III. RESEARCH METHODOLOGY

This narrative literature review was conducted in order gather both the problem and solution. For the analysis of literature, narrative method of review is used. This method of literature review on the usage of technology empowers the broader picture of problems and controversies associated with the use to technology (Frennert & Östlund, 2018). It helps in the analysis of debates, results of previously conducted research and current shortage of knowledge and at the same time, it also helps in suggesting the future implications (Ferrari, 2015).

A. Data Collection

The data used in the present research paper was Secondary in nature which was collected from the reports of nationally and internationally recognized organization i.e. WHO, world bank. The evidence from previously conducted research (queuing theory and discrete event simulation) were collected. Moreover, the collected data was organized in tables for the clear understanding. The most recent literature was reviewed so that the recent trends on the subject could be highlighted.

B. Data Analysis

All the data gathered and organized for the clear depiction of trends and broad understanding. For data organization, MS excel was used.

IV. LITERATURE REVIEW

In order to understand at the broad level, firstly, healthcare system was described in the general perspective and then it was carried out by focusing it in the context of Pakistani healthcare system (problems and ultimate solution by using queuing theory and discrete event simulation).

A. Healthcare system

Good health of the people is essential in order to develop and improve the economic state of country.Health care delivery among various service deliveries has been explained as the kind of delivery in which there is high consumer involvement in the process of consumption process. In whole of the process client/patient is involved. A bad service delivery can harm the patient and it may lead to the loss of his/her life.This is the reason, investigating patient/client satisfaction is necessary effort in order to bring about the improvement in the quality of health care system(Nkrumah et. al., 2015). Patient flow in the hospitals is particular interest of the researchers and that of practitioners, with the assumption that on improving the patient flow there would be significant impact on the patient satisfaction and so on the quality (Armony et. al., 2015). The perspective of patient satisfaction has gained serious attention in hospital care in the recent years (Khamis & Njau, 2014). Service quality and patient satisfaction are in a close relationship, good service quality gives out the encouragement to the patient to go for a strong relationship with the particular hospital (Surydana, 2017), (Kalwar, 2020). Service quality according to Kotler (2009) is the difference between real and expected of customers supposed to be provided to the customer (Surydana, 2017). Nowadays, patients bear many opinions

when deciding upon to have the services of particular healthcare provider (Yeddula, 2012). The image and patient satisfaction are important factor for companies to influence both tangible patient loyalty for re-coming and recommend it to others(Juhana et. al., 2015), (Kalwar et. al., 2018).

Hospitals are one of the most important links in the healthcare service chain and their quality directly affects people's lives (Dong, Yom-tov & Yom-tov, 2015). Hospital is the major section in the view of health care settings, it is found to have major impact on disease prevention, earlier detection, treatment and restoration of patients (Haghighinejad et. al., 2016). Globally, the main focus of hospitals is on occupancy and discharge rates of patients so that the executive capacity of the system can be figured out. Methods of Management Sciences or Operations Research can help managers who are involved in the activity of planning and management of resources (Gunal, 2012). Tremendous literature is available on the reforms of health care and many methods are debated regarding how the reforms can be assessed? An explicit framework is required for performance management against which the performance can be judged and quantified (Tandon et. al., 2002).

1) Healthcare System in Pakistan

In Pakistan, the distribution of healthcare facilities is unjust in public as well as private sector, by which they are made inaccessible to people of low income in rural areas (Naz et. al. ,2012), (Bergman, 2011). Pakistan is categorized as the country with low income (World Bank, 2006) and according to Human Poverty Index (HPI), it is ranked at 65th among 102 developing countries (Watkins, 2006). However, Human development Index (HDI) is upgraded from 0.346 as it was in 1975 to 0.539 in 2006, this is quite slow improvement and is ranked at 134th number in 2006 United Nations Development Program (UNDP) HDI due to its poor social and development indicators in comparison to the countries of same level of economic development (WHO, 2007). Health care services regulations and coordination is still in state of evolution in the context of structure, roles and responsibilities (WBG, 2015). Throughout last five decades, Pakistan is improving quite slowly in health sector as witnessed by its mentioned health indicators, strengths and weaknesses. Therefore, the government should take the initiative of improving it (Kurji, Premani & Mithani, 2016). Usually, public sector is regarded as providing more reasonable and evidence-based care. After review of literature systematically by S. Basu et al (2012), this claim is not supported that private sector is more efficient, accountable or more effective in terms of medical services than public sector, however, public sector is frequently lacking in timeliness and patient hospitality apparently (Basu, et. al., 2012). In Pakistan, there is the existence of two parallel health care systems i.e. public and private. The private sector was so small at the very first until majority of the people visited there for resolution of their medical problems and then businessmen transformed it into the hospitals after the passage of time (Bergman, 2011). It was indicated by the literature that people of Pakistan are not capable enough in terms of affordability towards the private healthcare; Because of increasing industrialization and population, environmental pollution is increasing day by day and the public healthcare facilities not sufficiently provided: therefore, it was suggested to Government in that it should focus on bringing about improvement in public healthcare (Watkins, 2006).

In Pakistani public sector hospitals, the quality of the healthcare service delivery is highly ignored and inconsistencies of the process are not perceived as the main problem in health care facilities (Sajid et. al., 2008). The patient`s visits to the hospitals are observed to be increasing with the increasing population because in the context of relationship between health and development Pakistan is at main intersection being the 6th highly populated country with the population of 191.71 million with growth rate of 1.91% (Mashhadi et. al., 2016). Therefore, due to continuously increasing population, health care services also need to be enhanced and improved so that the people can be provided better health care by the Government. Hospitals are the one of essential connections in the chain of health care service and the lives of the people are directly affected by the quality of healthcare (Gunal, 2012).

a) Problems of Healthcare in Pakistan

Public healthcare system in Pakistan is large and dispersed and is given in access to people with trained doctors, staff and medicines; but there was the problem of unavailability medicines and doctors due to the unavailability of doctors; It was concluded in the study that these both problems were due to managerial constraints not because of financial constraints (Callen et. al., 2013). An empirical study conducted to expose the problems faced by the patients in public health care hospitals. Study showed that 36.4% patients were poor who visited the hospital, 41.8% patients reported that staff is frustrated towards the patients; furthermore, 72.7% respondents had common opinion that poor patients are not well treated, whereas, 96.4% of patients reported about the preference of doctors to relatives and known patients (Ahmad et. al. , 2013). Another study was conducted on public sector hospitals of Pakistan, and it was revealed that mostly poor people visited the public hospital for their health issues and they faced variety of problems there in terms of treatment and facilities. The picture of public healthcare service delivery represents an even distribution of resources between urban and rural region. The poor in the rural areas are at obvious disadvantage in the context of primary and tertiary public healthcare facilities and they also fail to take advantage of immunization of their children from the public programs (Afzal & Yusuf, 2013). Due to poor health services patients especially children and women are suffering a lot. It be seen in the table 1 that the health indicators are poorly decreasing; after the period of two decades, infant mortality is decreased from 95/1000 lives to 60/1000 lives. Same case is with the maternal mortality rate, it has decreased from 490 -260 lives per 100,000 lives. Children mortality under five years is decreased from 122 – 74 in two decades.

There is greater number of children which are affected because of not being provided the proper vaccines. However, the report of World Bank Group 2015, shows that infant mortality rate is 69/1000 live births. It seems to be increased than that was reported by World Health Organization in 2013 i.e. 60.

TABLE I. HEALTH INDICATORS OF COUNTRIES OF SOUTH ASIA

Health Indicators	Pakistan		Bangladesh		India		Sri Lanka		Nepal	
	1990	2010	1990	2010	1990	2010	1990	2010	1990	2010
Infant mortality rate (per 1,000 live births)	95	60	97	39	81	49	24	11	94	41
Maternal mortality rate (per 1,000 live births)	490	260	800	240	600	200	85	35	770	170
Under-five mortality rate (per 1,000 live births)	122	74	139	49	114	63	29	13	135	50
Immunization (DPT)* among 1-year old (%)	54	86	69	95	90	72	86	99	43	82
Immunization (measles) among 1-year old (%)	50	82	65	94	56	74	88	99	57	86
Total fertility rate (births per woman)	-	3.4	-	2.2	-	2.6	-	2.3	-	2.7
Life expectancy at birth (years)	-	65.2	-	68.6	-	65.1	-	74.7	-	68.4

Source: World Health Organization (2013) (Breu, Guggenbichler & Wollmann, 2013)

This data is an accurate and proper reflection of poor health care services of public health sector of Pakistan. These issues must be resolved so that the existing healthcare system can be improved and vulnerable people can be provided better access to public healthcare facilities and service. The reasons behind poor health services at public hospitals are limited governmental funding, lack of governmental interest in launching new healthcare projects and over-burdened public hospitals (Ahmad et. al. , 2013).

TABLE II. HEALTH INDICATORS OF COUNTRIES OF SOUTH ASIA

	Infant mortality rate (per 1,000 live births)	Malnutrition prevalence, (% of children under age 5)	Maternal mortality rate (per 1,000 live births)
Country/Year	2013	2004-11	2013
Afghanistan	70	-	400
Bangladesh	33	42	170
India	41	48	190
Nepal	32	41	190
Pakistan	69	43	170
Sri Lanka	8	19	29

Source: The world Bank Group (2015) (WBG, 2015)

It was suggested in the research that the Government should focus on health care in terms of proper medical equipment and infrastructure maintain check and balance, in this way the problems can be reduced (Naz et. al. ,2012). The human resources for healthcare are gradually increasing in Pakistan year by year. As reported by International Finance Corporation (IFC) in its report of 2011 that, 5000 medical graduates are produced by different universities and colleges. The present ratio of doctor to person is 1:1,183, which is quite below than the standard recommended by World Health Organization (WHO) i.e. 1:1000 (Bergman, 2011). The requirement of medical personnel i.e. doctors almost complete; it is quite near to the standard recommended by World Health Organization. Now, dynamic leadership and governance is needed desperately for designing and enforcing evidence-based policies, programs and the way to take care of the system (Kumar & Bano, 2017).

Besides above mentioned problems the patients in the public sector hospitals face too many problems and delay is one of those major problem. Patients wait too long in order to get served in hospitals, it's a potential threat to healthcare services and is observed to an increasing extent (Obulor & E. B.O, 2016). Healthcare sector is facing the problem of delays. All of us wait for an appointment regarding health issue at hospitals and after arriving the facility we are supposed to wait even more to see the doctor. Delay is not unusual in hospitals, we always find may people awaiting at different stages in the hospitals i.e. patients waiting for surgery, Diagnostic tests, OPDs, Emergencies etc. (Green, 2006).

2) Problems of Hospitals

Nowadays, the big problems of the hospitals is congestion of patients at receptions, out-patient departments (OPDs), emergency departments, intensive care unit (ICU) and waiting areas. This is because of inaccuracy of planned queuing system in the various areas of hospitals. Queue is the common occurrence in daily life (Kembe et. al., 2012)–(Yusuff, 2015)e.g. at Hospitals. When the numbers of patients exceed the number of doctors then the queue is formed. Outdoor patient department (OPDs), emergency department (EDs) are the most visited departments at any hospital and are the initial confrontation of a patient with staff of hospital to get service(Wang et. al., 2009). Highlighted problem that the patients face is long queues which causes delay for patients to consult the doctor. Most of the patients die until their turn to get the service. OPDs and EDs play an important role in the healthcare services. In the past ten years, developed countries have stressed on the EDs in terms of overcrowding crisis and their impact on service time and on the same time great attention was paid on the ability of the hospital to meet medical emergency needs (Haghighinejad et. al., 2016). Due to

lacking in the control over the customer services, the capacity planning of the particular department can complicated by the service demands (Uriarte et. al., 2015).Delay is the difference between service demand and the available capacity to meet the demand (Green, 2011). Long waiting experience of the patients in the queue usually have negative impact on the satisfaction of the patient (Obamiro, 2010). Which incurs cost to organizations, which is termed as cost of customer dissatisfaction (Kembe et. al., 2012), (Agyei, Darko, and Odilon, 2015). Poor health services are acting as obstacles against the overall development of Pakistan (Mustafa, 2015).

B. Application of queuing theory for the improvement of healthcare operations

Hospitals are known as complex systems which are included with some societal benefits and bulk incurred costs. Those costs are made to be incurred more because of inefficacies of processes which occur due to congestion and delays in the patients care systems. Literature indicates that prediction of level of congestion and needed capacity is impossible to be figured out without the help of queuing models (Green, 2006). Therefore, in order to study and improve patients flow, it is suitable to look at the facility with the lens queuing network (Armony et. al., 2015). Queuing models are required to be put in a little data and results can be calculated by the help of simple formulae in terms of performance measures; this is an easier way to figure out the optimum solutions instead of estimating the performance of the system in the provided context (Green, 2006).

The most practical approach to solve these sort of issues is queuing theory or waiting line theory (Olorunsola et. al., 2014). It was developed by renowned Danish telephone engineer AgnerKrarupErlang in 1913 (Bastani, 2009),(Kissani & Rifai, 2015),(Green, 2011), (Mustafa & Nisa, 2015)–(Varma, 2016). He was the first scientist in 20th century who treated the congestion problem in the context of telephone exchange(Mwangi & Ombuni, 2015). The major elements in waiting line theory involve people getting services, entrance process, queue formation, discipline of the queues and the service mechanisms in the service industries like as hospitals, waiting times of the customers/patients must be predicted at the different levels of the service (Fitzsimmons, Fitzsimmons & Bordoli, 2008). Queue or waiting line at hospitals are associated with waiting cost of patients, when they are made to wait in the hospitals (Kembe et. al., 2012)(Khaskheli et. al., 2020). These problems are solved and simplified by using the queuing theory. In which waiting time and service times are calculated and also the optimum service level and waiting time of the patient can be calculated (Varma, 2016). The study of queue deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and system respectively and average facility utilization(Adaora, 2013).Queuing models are used to study queue systematically (Bastani, 2009), (Kandemir & Cavas, 2007). Due to dealing with the overcrowded scenarios, queuing theory is also known as the theory of overcrowding(Adaora, 2013). It is widely used in service organizations for waiting lines to be analyzed and their processes to be modeled (Olorunsola et. al., 2014). Aggressive driving behavior has become prevalent (Kalwar et. al., 2020), (Khaskheli et. al., 2018). In recent years it has been the teething concern in health care services (Ikwunne & Onyesolu, 2016). It is required to increase the customer satisfaction by reducing the queue and making service delivery efficient (Fomundam & Herrmann, 2007).The analytical approach i.e. queuing formulae and its solutions are only possible when the hospital already exists, then the data would be collected after the collection of data, the optimum queuing system can be suggested. What if the actual system is not installed and the scenario which is not real but the computer`s manipulated calculations are taken out by different software, which are considered as the replica of real world systems. This method is called as simulation, now a days most of the decisions are made on the basis of simulation. Suppose we are taking the simulation model of radiology department of the hospital, it can be used for the better understanding of the impact of new machine that it may have on the service quality of hospital (Gunal, 2012). Simulation is more effective as compared to analytical solutions because in case simulation analysis of complex models, the conditions can be changed and the behavior of model can be judged (Haghighinejad et. al., 2016).

1) Related Research Work

Mustafa &Nisa, 2015 used applications of queuing theory in healthcare were mainly focused. Different departments i.e. patient's registration department, outpatients department (OPD) and pharmacy were under consideration: on the same time, different processes in in the queuing system were also kept on the observation. Exponential and poison distribution were used for the service and arrival of patients respectively. Single server M/M/1 and multiple server queuing models M/M/2 were used for the calculation of performance measures and for analysis simulation was used. Furthermore, correlation among performance measures was also calculated (Mustafa & Nisa, 2015).

Olorunsola et al., 2014 modelled Patients` flow in his research; Queuing theory was used and the performance measures were calculated so that optimum bed count could be determined. For the analysis of queuing system multi-server queuing model was used. As per its assumptions, it was assumed that the arrival and service of patients followed the poison and exponential distribution respectively. Reneging logic was used for the analysis of customers`/patients` waiting times at Emergency and Accident Department of the respective hospital. On the same time, its impact on the number of beds was investigated in different departments (Olorunsola et. al., 2014).

Odunukwe, 2013 conducted study with the aim of determining the average time of customers they spent in queue and their time to get served by the service channel; then impact of wasted time could be investigated on the cost associated. Data were collected by considering the arrival pattern and service pattern of customers. Birth and death Markovian process was used; furthermore, chi-square test was used to confirm the arrival and service distributions if they were Poisson and exponential distributions. After this confirmation the data could be analyzed by using Markovian birth and death

process. The results indicated service rate is 0.1521 and arrival rate was 0.2157, the probability of servers to remain idle was 0.2786 (27%) and the waiting cost was calculated to be N938.597. In last, management was suggested to increase the number of servers to three so that the waiting time of patients and the associated cost could be minimized(Adaora, 2013).

Bastani, 2009 estimated the number of required beds at the hospital. This was accomplished by the use an adequate model by the help of which patient flow could be analyzed in and between the various departments of the hospital. Stream of emergency patients was only focused and the network was developed in which the interaction between the intensive care unit (ICU) and the monitor unit (MU) was modelled, which was meant to be causing a main proportion of the congestion in the emergency department (ED). MATLAB was used to develop and simulate the model. It was concluded that the approximate number of beds was 14 in the Intensive care unit ICU and 208 beds in the MU for the new hospital setup which was under consideration(Bastani, 2009).

McManus et al., 2004 developed a mathematical model for the patient flow. Data of the patients i.e., admission and discharge of the patients in ICUs. By using queuing theory application a mathematical model of patient flow was developed. The actual/real scenario was compared with the results of the model. After the validation the model was proved to be accurate(McManus et. al., 2004).

Obamiro, 2010 used multi-server queuing model to carry out the study. In this research pregnant women arrival was focused in the hospital. Data was collected for the month. The frequency of the pregnant women and their waiting times were evaluated in the different weeks of the month. For data analysis TORA software was used(Obamiro, 2010).

Kembe et al., 2012 conducted his research at Riverside hospital. In this study multi-server model was used to calculate the waiting cost, service cost and opportunity cost and total system cost by using the performance measures of the queuing system. TORA software was used to analyze the data. In conclusion it was suggested that management of the hospital should increase the number of doctors from 10 to 12(Kembe et. al., 2012).

Puoza&Hoggar, 2014 used multi-server queuing model to resolve the issue of long waiting times of patients in the queue and over utilization of doctors. QM software was used to calculate the all performance measures and parameters of the queuing system (Puoza & Hoggar, 2014).

Mensah &Asamoah, 2014 used single server queuing model to analyze the data. Two hospitals i.e., Nkawie Government Hospital and Aniwaa Medical Centre which is fully private owned were compared in terms of traffic intensity and utilization factor. The data was analyzed in QM software version 2.2(Mensah & Asamoah, 2014).

Armony et al., 2015 studied the flow at Israeli hospital through the exploratory data analysis. There were certain questions that were raised by exploratory data analysis (EDA) i.e. 1) Can a simple queuing model usefully capture the complex operational reality of the emergency department (ED)? 2) What time scales and operational regiems are relevant for modeling patient length of stay in the internal ward (IWs)? 3) How do protocols of patient transfer between the ED and IWs influence patient delay, workload division and fairness? Relating bottlenecks from ED to IW physician protocols were also given importance(Armony et. al., 2015).

Varma, 2016 aimed to reduce the waiting time of patients, they spent while in remaining in the clinic. Single server queuing model was used to calculate the waiting time of patients, traffic intensity, and average number of individuals in the system. Poison arrival of patients based on first come first served discipline and exponential distribution service rate were the assumptions(Varma, 2016).

Ikwunne&Onyesolu, 2016 used multi server queuing model to find the optimum service level. The author has suggested the optimum number of doctors, so that the waiting time of patients can be minimized. Production management and operations management (POM QM) and queuing theory calculator was used to analyze the data(Ikwunne & Onyesolu, 2016).

Khaskheli et al., 2020 conducted their research to suggest the optimum number of receptionists and doctors at the study areas in order to optimize the performance of existing queuing systems at the out-patient departments (OPDs). The most congested OPD i.e. medical OPD was selected for the study at the case hospital 1 and then same OPD was selected in another public sector hospital (case hospital 2). Both hospitals were the tertiary care hospitals of the different districts of Sindh Pakistan. Data was collected for two weeks: data collection parameters were; arrival rate, service rate of patients, number of servers, salaries of the servers and associated waiting cost of patients. Arrival and service distribution of the patients were verified as per assumptions of the multi-server queuing model (M/M/c) by using input analyzer of Rockwell Arena 14.5. Performance measures of the queuing system were calculated by using TORA optimization software. For cost calculation and graph plots MS excel was used. According to the results, one receptionist and doctor was suggested to be increased at both of the OPDs for the minimization of congestion of patients and their waiting times (Khaskheli et. al., 2020).

Unlike queuing theory, the simulation approach is more simple and detailed. Simulation leads to the clear understanding of where bottlenecks exist in the queuing system. Simulation is a mimic of reality that exists or is contemplated. Simulation is most effectively used as a stage in queuing analysis. The simulation is run for patients coming to department, the pertinent parameters like waiting time, service time, waiting time-service time ratio (Lade, Choriwar & Sawaitul, 2013). It is an analytical tool used for creation, maintenance, evaluation or improvement of a system or process. In fifties it was firstly used in the healthcare operations, it was used in order to increase the efficiency of healthcare operations and after that it has been used as powerful tool for improving and analyzing the healthcare systems (Uriarte et. al., 2015).

C. *Application of modeling and simulation for the improvement of healthcare operations*

Real world is represented quantitatively by the help of discrete event simulation (DES), by which its dynamics is simulated on an event-by-event basis, and detailed performance report is generated (Babulak & Wang, 2010). There are three most common computer simulations i.e. discrete event simulation (DES), system dynamic simulation (SDS), and Monte Carlo simulation (MCS) used in health care: the most popular method is discrete event simulation(Hong et. al., 2013). Queuing models for systems are inflexible unless numbers of simplifying assumptions are made. Because of these reasons, queuing systems are complex and cannot be analyzed typically by using mathematical models. Instead, discrete-event simulation is used, in which the values and expressions of different probability distributions i.e. arrival and service distribution and on the same time it keeps track of relevant statistics, which is used to analyze such systems and calculate the performance metrics (Chan & Green, 2013). In different clinical settings, researchers of operations research use queuing theory and discrete event simulation techniques and use them for deciding the different appointment strategies. (Priyan, 2017). In the early duration of its development, discrete event simulation was used greatly in the manufacturing sector: a basic structure i.e. active and dead stage is followed by these models where the customers move from an activity to the queue and usually changing between these two components. (Swinerd & McNaught, 2014). Organizations are needed to improve their processes and practices at the advent of new technology in the market (Kalwar & Khan, 2020)–(Arain, Khan & Kalwar, 2020). Technologies of discrete event simulation (DEs) have been greatly used by industry as well as academia in order to deal with different industrial problems (Babulak & Wang, 2010). It has greatly grown out of modeling manufacturing systems, nowadays it has increasingly been applied in the service sector (Swinerd & McNaught, 2014). A research was conducted in the 1970s in which DES was used) to improve flow of patients in emergency rooms and office doctor (Roberts, 2015). This technique allows end users (i.e. hospital administrator, clinic manager) to assess the efficiency of existing health care delivery system(Aeenparast et. al., 2013).

1) *Related Research Work*

Kalwar et al., 2020 aimed to suggest the optimum number and schedule of doctors at the OPD (Out-Patient Department) of Gastrology of a hospital in Pakistan. In order to achieve this aim, the discrete event simulation model is developed to minimize waiting time of patients. Data is collected for one week from the OPD; Data collection variables are arrival and service rate of patients, their salaries/income, patient‘s OPD fee, doctor’s charges/patient, service time of patients at each of service channel i.e. reception, triage and doctors’ cabin. Stop watch is used for recording the service time of patients. Input analyzer is used to reveal the distribution of the data. Rockwell arena software version 14.5 is used to model and simulate the queuing system of the outpatient department. Scenario analysis is conducted in four scenarios; in each of the scenario doctors were assumed to be seated for one additional hour. During the period of data collection, it is observed that most of the patients are coming with an appointment of doctors therefore, it is not justified to suggest the hiring of new doctor; especially when patients are coming for the particular doctor; therefore, already available doctors are suggested to be seated longer in the OPD; that is the way to serve the maximum number of patients in the virtual queue of patients that has been kept waiting for having an appointment and for their turn to see the doctor(Kalwar, 2020).

Roumani, 2013developed discrete event simulation model with the help of Arena software version 13.0. The model was simulated for 10000 days with the exercise 50000 days. The period which is called warm-up period is necessary because it ensures that whether the system has reached to the steady state or not before any statistics is recorded. It was executed twice, at the first execution the Markovin arrival and service were assumed. Secondly it was assumed generally and exponentially distributed service was assumed. For validation of simulation model, the actual flow of patients was compared with the simulation results of the model (Roumani, 2013).

Connelly & Bair, 2004explored the potential of discrete event simulation (DES) methods to advance system level investigation of emergency department operations. Development and operation of emergency department simulation was described new platform for computer simulation activity at a level 1 trauma center. The extend DES modeling package was used to develop the model. The inputs of the model were staffing level, facility characteristics and patient data drawn from the electronic databases. The accuracy of the model was tested by comparing predicted and known patients service times(Connelly & Bair, 2004).

Lade et al., 2013conducted his research with the objective of minimization of patient waiting times in different sections of radiation therapy and oncology department. In order to achieve the set objectives, simulation was carried out by focusing 59 patients. Analysis revealed that the doctors should be increased from 4 – 5; After the increment of one doctor the average waiting time of patients decreased from 7.20 to 4.25 minutes(Lade, Choriwar & Sawaitul, 2013).

Uriarte et al., 2015conducted his research based on discrete event simulation based at Swedish Emergency department by applying Multi-Objective Optimization simulation technique in order to study the system improvement analysis. Number of solution were provided to the decision makers after the analysis was done, in which the length of stay and waiting times of patients were reduced for Emergency department. Multi-Objective Optimization simulation technique was proved to be useful technique for improving healthcare processes(Uriarte et. al., 2015).

Dawei, 2009conducted simulation to analyze the process of outpatient service, simulation as conducted by using the Arena simulation software. The current process of services was compared with the simulated process so that bottlenecks can be identified. It was revealed that patients were spending 7.2% of their total times in the OPD uselessly. This problem was due to the saturation of medical resources. Therefore, it was suggested that the outpatient resources should be reasonably coordinated in order to improve the outpatient flow(Dawei, 2009).

Wang et al., 2009constructed a model for emergency services by using ARIS and Arena software. Two assumed scenarios were tested in these software. The model was constructed in order to rectify the bottlenecks in the system and optimally allocated the resources at the different stages of services(Wang et. al., 2009).

Haghighinejad et al., 2016aimed to reduce the waiting time of the patients they wait in the queue and emergency department of Iranian hospital. Arena version 14 was used for the simulation of results. There was the issue of bed capacity in the emergency department, and because of that reason patients waited too long to get served. Therefore, it was suggested in the study that number of beds should be increased from 81 to 179. Classification of patients was defined in terms of priority; First priority was given to patients with the high degree of illness(Haghighinejad et. al., 2016).

Kanagarajah et al., 2008presented architecture of complex systems and an agent based modeling framework for studding the improvements in healthcare system and their influence on patient safety, workloads and economics. The application of a safety dynamics model proposed by Cook and Rasmussen is demonstrated in order to study and analyze the healthcare system by using simulation of an emergency department hypothetically. By means of simulation, complexities of healthcare system and its nonlinear behaviors of is demonstrated in this paper; this model in various aspects of healthcare was evaluated with the question of how it can be used in healthcare setting. Its societal, organizational and operational consequences were evaluated in assumed scenario of its application in healthcare setting(Kanagarajah et. al., 2008).

Kittipittayakorn& Ying, 2016integrated discrete event simulation and agent based simulation so that the waiting time of patients could be minimized. Patient`s behavior was modelled from the collected data and also were incorporated in the agent based simulation. Proposed approach was an aid for the analysis and modification of processes of orthopedic department and it provides the more reliable results by considering more details. After the implementation of proposed approach, the total waiting time of patients at orthopedic department reduced from 1246.39 minutes to 847.21 minutes(Kittipittayakorn & Ying, 2016).

Nunez-Perez et al., 2017designed operational strategies and pretested for better care delivery at emergency department by the use of discrete event simulation. At the very first, input analysis of was carried out. Then the simulation model was developed and validated in order to reveal that whether it coincided with the real world or not. After the development of model, performance indicators were calculated and their analysis was conducted. In last the strategies for the improvement were suggested after the evaluation through the modeling, simulation and statistical analysis. It was demonstrated by the results that waiting time of patients could be meaningfully reduced based on the suggested approaches in this research(Nunez-Perez et. al., 2017).

Shakoor et al., 2017improved the healthcare delivery at MRI by utilizing the Arena Simulation software. Radiology department of public hospital was selected for the conduction of this research. This was model based research and the main criteria used for the development of simulation model was service quality, medical needs of patients and the service cost. Data of arrival and process time was collected for the duration of one year. Model was developed in Arena simulation software. Results indicated that the resources at the research site were exhausted(Shakoor et. al., 2017).

Al-Araidah et al., 2012developed discrete event simulation model at local hospital. Data was collected form the ophthalmology outpatient clinic. Total time spent by them in the system was recorded along with the service time of patients then was fed in the discrete event simulation model. The developed model was compared statistically with the real scenario. Many alternative improvements were suggested after investigating through the discrete event simulation model. Key performance measures i.e. expected waiting time, expected visit length were traced in the model. Number of alternatives were found to be causing the waiting time reduction up to 29% and length of visit up to 19% after implementation(Al-Araidah, Boran & Wahsheh, 2012).

Maull et al., 2009analyzed the influence of fast track strategy (FTS) on patient waiting time in emergency department at the hospital. Discrete Event Simulation (DES) was used to develop the simulation model in order to predict output in a various categories of triage and comparison between these and post-implementation results was also conducted. Results indicated a significant reduction in waiting time of patients: 13.2% of the population was waiting more than four hours before implementation and it was compared with 1.4% after the implementation(Maull et. al., 2009).

Montgomery & Davis, 2013conducted his study by using discrete event simulation (DES), by the help of which the range of possible variables could be forecasted. Discrete event simulation (DES) permitted the incorporation of multi-layered variation by the help of probability distributions in the hospital; which helped in the determination of attributes of patient and their actions flowing through the system. Various scenarios were designed in order to determine the influence of closing or opening beds and changing policy of flow on the average daily census and number of beds occupied in the different sections and for the hospital. The results of this research indicated that how the decisions of information leaders might influence the system wholly and the long-term consequences of policy changes. The model of patient flow at the hospital was successful in creating virtual environment and permitting it to experimentation and therefore, mitigating the risk of investing resources in non-value added policy(Montgomery & Davis, 2013).

Mistakes or flaws cannot be overcome or improved until and unless they are detected. Simulation is a practical and justified approach to detect the bottlenecks in the queuing system and after the problems are detected they can be simplified. So in the context of public healthcare facilities of Pakistan it is highly required to investigate the queuing system of public sector hospitals, so that the patients may not face so many problems which are caused by the mismanagement and misallocation of medical resources.

V. SUMMARY OF LITERATURE

In the number of healthcare facilities, application of queuing theory and discrete event simulation were found to be used for the improvement. Literature indicates that queuing theory was used for the calculation of expected service cost, expected waiting cost, bed capacity at ED, ICU and MU etc. (see table 3). Table 3 shows the research which has been conducted in the field of health care operations and the improvements carried out in them.

TABLE III. SUMMARY OF LITERATURE REVIEW

Author(s)	Healthcare Facility	Analysis/Finding
Mustafa &Nisa, 2015	Reception, OPD and Pharmacy	Correlation analysis was conducted among the performance measures of queuing system by using M/M/1 and M/M/2 queuing models
Olorunsola et al., 2014	Emergency and accidents departments	Optimum number of beds was calculated and its effects on the other departments was also calculated
Odunukwe, 2013	OPD	Waiting time and cost of patients was minimized by increasing the number of servers at the facility.
Bastani, 2009	Intensive care unit and monitor unit	Number of beds were estimated for ICU and MU section for new setup of the hospital by the development of simulation model in MATLAB.
McManus et al., 2004	ICU	Mathematical model was developed for the patients` flow (admission and discharge) at the ICU.
Obamiro, 2010	OPD	Waiting time of pregnant women was analyzed by using multi-server queuing model in TORA optimization software.
Kembe et al., 2012	OPD	Patients` service cost and opportunity cost were calculated by using multi-server queuing model in TORA optimization software. In order to minimize the waiting cost of patients, the number of servers were increased.
Puoza&Hoggar, 2014	OPD	Long queues and utilization of doctors were studied and for analysis, QM software was used.
Mensah &Asamoah, 2014	Hospital	Traffic intensity and utilization factor of two hospitals were compared by using single server queuing model.
Armony et al., 2015	Emergency department (ED) and internal wards (IW)	The delay of patients transferred from ED to IW was analyzed and bottlenecks were highlighted by using the technique of exploratory data analysis.
Varma, 2016	Clinic	Patients waiting time, traffic intensity and average number patients were calculated by using single server queuing model. Moreover, he minimized the patients` waiting time at the facility.
Ikwunne&Onyesolu, 2016	Clinic	Optimum service level was calculated by using multi-server queuing model in Production management and operations management (POM QM) software
Khaskheli et al., 2020	Hospital	Compared the performance measures of two public sector hospitals (calculated by using multi-server queuing model in TORA optimization software). Waiting time of patients was also minimized by increasing the number of doctors.
Kalwar et al., 2020	OPD	Conducted discrete event simulation (DES) for the minimization of virtual queue of patients awaiting for healthcare service by using Rockwell Arena software.
Lade et al., 2013	Section of radiation therapy and oncology department	Waiting time of patients was minimized by increasing one doctor.
Dawei, 2009	OPD	The existing service of OPD and results of simulation were compared and it was revealed that the patients were spending extra useless time at the OPD.
Wang et al., 2009	ED	Optimal allocation of resources at the various level was determined by using ARIS and Arena software.
Haghighinejad et al., 2016	ED	Waiting time of patients was focused to be minimized at ED and it was highlighted that patients were waiting more at ED because of bed capacity problem.
Kanagarajah et al., 2008	ED	Simulation modelling framework was developed in order to study the advancements in the healthcare systems and their impact on the economics, workload and safety of patients.
Kittipittayakorn& Ying, 2016	Orthopedic department	DES and agent based simulation (ABS) were integrated for the purpose of minimization of patients` waiting time. In order to fulfil the purpose of the study, behavior of patients was modelled in ABS model.
Nunez-Perez et al., 2017	ED	DES was used to design the operational strategies and conducted a pretest health care delivery system in context of ED.
Shakoor et al., 2017	Radiology Department	DES was used to improve the healthcare delivery system at MRI. After the data collection and analysis, it was indicated that resources were exhausted.
Al-Araidah et al., 2012	ophthalmology outpatient clinic	DES model was developed in which expected waiting time and expected visiting time of patients was traced and numerous alternatives were found by the help of which patients` waiting time could be minimized.
Maull et al., 2009	ED	DES model was developed for the analysis of fast tract strategy in the patients` waiting time at ED. Results indicated a significant minimization of waiting time.

Since, ED, ICU and OPD are the facilities which were carried out in the previous research for the improvement in terms of their queuing systems. Since, the waiting time of patients is a factor worth working for because significant amount of cost is associated with it. Most of the researchers have calculated and worked on the patients waiting time at different healthcare facilities.

VI. Discussion

To improve population`s health status is the purpose of health care services(Lavanya & Ahmed, 2015). The activities or practices which are used for determination of health, health care delivery system (HCDS) is societal response to those activities. It is combination of people, agencies, organizations and number of resources by which healthcare system render its services by the help of them (Kumar & Bano, 2017), (Musgrove et. al., 2000). The efficient allocation of resources and channels are extremely necessary in order to provide quick and timely healthcare service to the patients at the every stage at healthcare facility. This is the reason, healthcare delivery systems has gained the attention of researchers since so long. Present research paper was aimed to present the broader and clear picture of healthcare problems in Pakistan and the various solutions are also discusses which are given by the researcher across the globe.

Delay occurs when there is less available capacity to meet the service demand (Green, 2011). When patients experience long waiting lines at the healthcare facility, it result in the dissatisfaction of patients (Obamiro, 2010). In this regard, the patients` waiting time to see the doctor must be minimized by making the procedure of hospital simpler and at the same time, they should be provided guidance by signboards for various departments (Lavanya & Ahmed, 2015). In patient flow is considered to be major element for the improvement of efficiency of healthcare services (Olorunsola et. al., 2014). There are certain frameworks which provide understanding for quantitative analysis of patient flow at emergency department, their waiting time, serving time and it also provides the analysis tools for analysis of factors which have an impact on mentioned outcomes (Connelly & Bair, 2004). There is another famous technique which has been used for the solving the problems of waiting lines known as queuing theory. A little data is supposed to be put into queuing models and in the form of performance measures they give result about the performance of the unit and accordingly, the optimal solution is calculated (Green, 2006). Queuing models are used for modelling process (which include waiting line) at the engineering industries (McManus et. al., 2004).

Mensah et al., 2014 conducted their research on the comparison of queuing systems of two hospitals i.e. Nkawie Government Hospital and Aniwaa Medical Centre (Private hospital). Government hospital was indicated to be with greater waiting times of patients in the comparison of private hospital although the figures of waiting times were same for both hospitals but on the basis of OPD cases government hospital (5040) and private hospital (8991). After such traffic at the private hospital, the traffic intensity and utilization factor were calculated to be the same for both hospitals. At the same time, it was also noted that waiting time of patients at government hospital was growing longer as compared to the private hospital. It was finally indicated that longer patients` waiting times at government hospital were because of inadequacies in supervision at the different levels of the services (Mensah & Asamoah, 2014). Kembe et al., 2012 analyzed three costs (service cost, waiting cost and total system cost) in order increase the number of doctors at riversite hospital. Where the total system cost was minimum he suggested to increase the number of doctors from 10 to 12 and the solution was counted to be optimum (Kembe et. al., 2012). Since, ICU is the critical care unit of any hospital; one of the researchers reported the array of factors that have an impact on ICU occupancy i.e. weekly and monthly variations in patients; moreover, he also reported the unit level factors i.e. patient case mix, size of unit and throughput of unit; in the factors which are considered as external factors i.e. size of the hospital, step down facilities at the hospital, models of care and practices for bed management (Tierney & Conroy, 2013). McManus et al., 2004 used the queuing model in his analysis and the model was proved to be accurate. The model was found useful for the prediction of monthly responsiveness to the varying demand. He conducted the correlation analysis among the various performance measures of the model (McManus et. al., 2004). Mustafa and Nisa, 2015 used queuing models and simulation in three departments of government hospital of Rawalpindi. They used both single server and multi-server queuing models and greater waiting time of patients was indicated by the model at pharmacy in the comparison of other departments. It was observed that waiting time of patients could be minimized by the help of multi-server queuing model.

VII. Conclusion

Mismanagement of the resources and the queuing system was highlighted to be the main reason for low quality of the healthcare service delivery in public sector hospitals of Pakistan. Behavior of staff with the arriving patients was reported to be irritating in public sector. Moreover, delayed service, long waiting times and less departmental capacity (at emergency, OPDs and laboratories) are the problems faced by the patients. On the same time, number of doctors is also less than required. Applications of queuing theory can bring a big improvement in the healthcare system. Problems related to the queuing system (i.e. patients` waiting time in the queue, delayed service at the end of hospital, bed capacity in wards, overall patients` flow/congestion) can be solved by the help of queuing theory (i.e. single server queuing model, multi-server queuing model by using TORA and POM software). Most importantly, there is another technique for mentioned problem. It is called discrete event simulation (DES). Simulation is replica of reality in which the existing systems can be modelled and analyzed by changing the various parameters simultaneously.Occurrence of waiting lines will always be prevalent in the systems (Brahma, 2013) and because of long waiting lines, doctors are prone to the stressful situation of examining patients in such a greater number and they strive to get free without examining them in deep. (Obamiro,

2010),(Yusuff, 2015), (Puoza & Hoggar, 2014), (Albert, 2007). In this situation, public health will certainly be put to an alarming condition; so to get rid of this situation, it is highly need to manage the queuing system in the most possible way. If the queuing system is aligned as optimum, no resources will be stuck in the system. Therefore, queuing simplification is the optimum solution in order to get out of congestion and waiting line problems.

VIII. RESEARCH GAP

After the deep review of literature, it was clear that none of the literature reviews discussed the healthcare problems in the respective country and the review of contribution of related methodology at the same time. The contribution of present research in highlighting the problems of healthcare delivery system in Pakistan and review of queuing theory and queuing simulation cannot be ignored.

IX. FUTURE IMPLICATIONS

When the system (healthcare facility) is congested, patients wait more in the queues and system in order to get served. AT ICUs and EDs patients are in critical conditions and if the patients are made to wait in that condition, anything can happen. In this regard, it is suggested that review of problems of ED and ICUs should also be reviewed specifically so that the problems can highlighted for the greater good of nation.

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XI. CONFLICT OF INTERESTS

There are no conflicts of interest among the authors of present research paper.

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