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Evaluation of Load Carrying Ability of Multilayer Cylindrical Shell

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Abstract- A method for an evaluation of thin-walled constructions fabricated of a composite material, a structure of which is a set of unidirectional reinforced layers of various orientations. A multi-layered hollow cylinder fabricated of a transversal-isotropic material with a given reinforcement structure is considered. It is demonstrated that the method, which is offered in this work for a calculation of load-carrying ability of multi-layered constructions, yields satisfactory results, which well agree with experimental data.

Keywords- *composite material, elastic characteristics, stress, multi-layered cylinder strength*

I. Introduction

Results of researches [1] indicate that a characteristic damage of multi-layered composites with a transverse-longitudinal and a quasi-isotropic placement of $[0, 90]_s$, $[0, 90, \pm 45]_S$ layer types under conditions of tension is a formation of crack massive oriented at an angle to a direction of load action. As a rule, the formation of crack network occurs long before a total break of construction. In this case, a hardness of composite material decreases and fibres breaks, which are initiated by cracks in a matrix, decrease the load-carrying ability and a service term of constructions fabricated of multi-layered materials. It is known that researches of reinforced multi-layered material strength are based on two approaches: a structural and a phenomenological one. As it was noted in [2], a modern state of structure approach to strength researches, which is employed in a micro-mechanical theory, does not allow reliable quantitative data for an evaluation of the composite strength. An analysis of strength criteria limitations and a description of destruction processes occurring in various composite materials are presented in fundamentals works [2, 3, 4]. As it was noted, a concentration of interlayer normal and tangent stresses near cracks at interface boundaries initiated a layering in adjacent regions. As a rule, to predict a layering initiation moment, all components of a three-dimension stressed state for a considered region of multi-layered composite are determined and the obtained values are substituted into the corresponding strength criteria. The layering is the most dangerous type of damage affecting a load-carrying ability of constructions, which are fabricated on the basis of composite materials. A number of known publications, which considered this problem [1], reported that researches of conditions of layering origination and the related stress redistribution were insufficient.

II. STRESSED STATE OF MULTI-LAYERED COMPOSITES WITH INTERFACE DEFECTS OF MATERIAL STRUCTURE

Characteristic features of composite materials with a layered structure are a high strength in a direction of reinforcement and a low resistance to an in-plane shear and a transverse break**.** A unidirectional layer is a construction element of multi-layered plates and shells. Both in the process of exploitation and at a moment of its fabrication, the composite strength depends on a normal stress of transversal direction and a tangent stress of interface shear. In addition, the unidirectional layer strength towards the reinforcement is essentially higher than its strength directed perpendicular to the reinforcement. Thus, for example, a uniaxial tension of three-layer material, extreme layers of which were reinforced in a tension direction and a center was reinforced in an orthogonal direction (Fig. 1a), was considered in [2]. Every layer represented a unidirectional reinforced material.

It was believed that the thickness of the outer layers of the same, i.e. $h^{(1)} = h^{(3)}$, and strain ε_{11}, ε_{22} for all segments of the same. In the case of plane strain when $\varepsilon_{22} = 0$, stress and strain in the fibers in the direction of the OX axis is found from the equilibrium condition

$$2\left(\sigma_{11}^{(1)}\bar{h}^{(1)} + \sigma_{11}^{(2)}\bar{h}^{(2)}\right) = \sigma_{11}$$

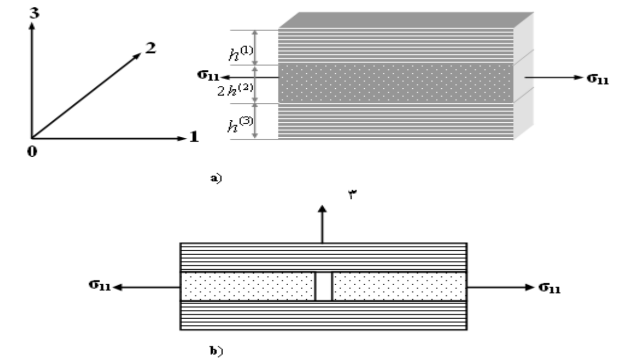

Figure 1 The structure of a three-layer composite material

The following relations:

$$\sigma_{11}^{(i)}=\frac{\sigma_{11}E_i^{(i)}}{2\left(E_1^{(1)}\bar{h}^{(1)}+E_2^{(2)}\bar{h}^{(2)}\right)}\qquad (i=1,2), \tag{1}$$

$$\varepsilon_{11}^{(i)}=\frac{\sigma_{11}}{2\left(E_1^{(1)}\bar{h}^{(1)}+E_2^{(2)}\bar{h}^{(2)}\right)},\qquad \varepsilon_{22}^{(i)}=0\quad (i=1,2). \tag{2}$$

Here $E_1^{(1)}$, $E_2^{(2)}$ – elastic module of a unidirectional material, respectively, in the longitudinal and transverse directions; $\bar{h}^{(i)}=\frac{h^{(i)}}{2(h^{(1)}+h^{(2)})}$ $(i=1,2)$ – the relative thickness of layers. A characteristic feature of modern unidirectional composite materials - a noticeable difference between the rigidity of such materials along and across fiber reinforcement. The obvious conclusion is that the destruction of the material considered (Fig. 1 b) begins with the second layer. As follows from formula (1), the destruction of this layer occurs at stress σ_{11} equal

$$\sigma_{11}^{*}=2\frac{\sigma_{22}^{(2)+}}{E_2^{(2)}}(E_1^{(1)}\bar{h}^{(1)}+E_2^{(2)}\bar{h}^{(2)}), \tag{3}$$

Where $\sigma_{22}^{(2)+}$ – tensile strength of the material in the transverse direction for the middle layer in tension. Solution to the problem [2] with the additional stresses caused by the formation of cracks is determined by the relations:

$$\sigma_{11}^{(1)*}=\sigma_{11}^{(1)}+\sigma_{11}^{(2)}\frac{h^{(2)}}{h^{(1)}}e^{-k_1x}\left(\frac{k_1}{k_2}\sin k_2x+\cos k_2x\right),$$

$$\sigma_{11}^{(2)*}=\sigma_{11}^{(2)}\left[1-e^{-k_1x}\left(\frac{k_1}{k_2}\sin k_2x+\cos k_2x\right)\right],$$

$$\sigma_{13}^{(1)*}=-\frac{h^{(2)}}{h^{(1)}}\left[z-\left(h^{(1)}+h^{(2)}\right)\right]\frac{\sigma_{11}^{(2)}}{k_2}\left(k_1^2+k_2^2\right)e^{-k_1x}\sin k_2x,$$

$$\sigma_{13}^{(2)*}=\frac{\sigma_{11}^{(2)}}{k_2}\left(k_1^2+k_2^2\right)z\cdot e^{-k_1x}\sin k_2x,$$

$$\sigma_{33}^{(1)*}=\frac{h^{(2)}}{2h^{(1)}}\left[z-\left(h^{(1)}+h^{(2)}\right)\right]^2\frac{\sigma_{11}^{(2)}}{k_2}\left(k_1^2+k_2^2\right)e^{-k_1x}\left(k_1\sin k_2x-k_2\cos k_2x\right),$$

$$\sigma_{33}^{(2)*}=-\frac{\sigma_{11}^{(2)}}{2}\left[z^2-h^{(2)}\left(h^{(1)}+h^{(2)}\right)\right]\frac{k_1^2+k_2^2}{2}e^{-k_1x}\left(k_1\sin k_2x-k_2\cos k_2x\right). \tag{4}$$

Parameters k_1 and k_2 equal

$$k_{1,2}=\sqrt{0{,}5(b^2\pm a^2)} \tag{5}$$

Where

$$a^2=\left[\frac{h^{(2)3}}{3G_{23}^{(2)}}+\frac{h^{(2)2}h^{(1)}}{3G_{13}^{(1)}}-\frac{\nu_{13}}{E_2^{(2)}}\left(2\frac{h^{(2)3}}{3}+h^{(1)}h^{(2)2}\right)+\frac{\nu_{12}}{3E_1^{(1)}}h^{(1)}h^{(2)2}\right]/A,$$

$$b^4=2h^{(2)2}\left(\frac{1}{E_2^{(2)}h^{(2)}}+\frac{1}{E_1^{(1)}h^{(1)}}\right)/A,$$

$$A=\frac{1}{2E_2}\left[\frac{h^{(2)5}}{5}-\frac{2h^{(2)4}}{3}(h^{(1)}+h^{(2)})+h^{(2)3}\left(h^{(1)}+h^{(2)}\right)^2+\frac{1}{5}h^{(1)3}h^{(2)2}\right].$$

In this case, axis ox coincides with the axis 01 analysis of the expressions (4) – (5) show that the stress $\sigma_{33}^{(2)*}$ и $\sigma_{13}^{(2)*}$ decay rapidly with distance from the edge of the crack and when $x=2\pi/k_2$ practically zero. A similar distribution pattern are the stress $\sigma_{11}^{(2)*}$, when $x=2\pi/k_2$ asymptotically approaches the value $\sigma_{11}^{(2)}$. Thus, the maximum stress $\sigma_{11}^{(2)*}$ occur at a distance π/k_2 from the edge of the crack:

$$\max\ \sigma_{11}^{(2)*}=\sigma_{11}^{(2)}\left(1+e^{-\frac{\pi k_1}{k_2}}\right). \tag{6}$$

Stress state of a single block of the middle layer length π/k_2 between adjacent cracks is described by the following expressions:

$$\sigma_{11}^{(1)*}=\sigma_{11}^{(1)}+\frac{\sigma_{11}^{(2)}h^{(2)}}{h^{(1)}\mathrm{sh}\frac{\pi}{2}\frac{k_1}{k_2}}\left(\frac{k_1}{k_2}\mathrm{ch}k_1x\cos k_2x+\mathrm{sh}k_1x\sin k_2x\right),$$

$$\sigma_{11}^{(2)*}=\sigma_{11}^{(2)}\left[1-\frac{1}{\mathrm{sh}\frac{\pi}{2}\frac{k_1}{k_2}}\left(\frac{k_1}{k_2}\mathrm{ch}k_1x\cos k_2x+\mathrm{sh}k_1x\sin k_2x\right)\right],$$

$$\sigma_{13}^{(1)*}=\frac{h^{(2)}}{h^{(1)}}\left[z-(h^{(1)}+h^{(2)})\right]\frac{\sigma_{11}^{(2)}(k_1^2+k_2^2)}{k_2\mathrm{sh}\frac{\pi k_1}{2k_2}}\mathrm{sh}k_1x\cos k_2x,$$

$$\sigma_{13}^{(2)*}=-\frac{\sigma_{11}^{(2)}(k_1^2+k_2^2)}{k_2\mathrm{sh}\frac{\pi k_1}{2k_2}}z\,\mathrm{sh}k_1x\cos k_2x,$$

$$\sigma_{33}^{(1)*}=\frac{h^{(2)}}{2h^{(1)}}\sigma_{11}^{(2)}\left[z-(h^{(1)}+h^{(2)})\right]^2\frac{(k_1^2+k_2^2)}{k_2sh\frac{\pi k_1}{2k_2}}$$

$$(k_1chk_1x\cos k_2x-k_2shk_1x\sin k_2x),$$

$$\sigma_{33}^{(2)*} = -\frac{1}{2}\sigma_{11}^{(2)}\left[z^2 - h^{(2)}(h^{(1)} + h^{(2)})\right]$$

$$\frac{(k_1^2 + k_2^2)}{k_2 sh\frac{\pi k_1}{2k_2}}(k_1 chk_1 x \cos k_2 x - k_2 shk_1 x \sin k_2 x). \qquad (7)$$

III. MODIFIED STRENGTH CRITERIUM FOR MULTI-LAYERED COMPOSITE WITH STRESS POINT AT LAYER INTERFACE

The most general formulation of the criterion of strength of anisotropic bodies is of the form

$$\left(R_{ij}\sigma_{ij}\right)^{\alpha} + \left(R_{ijkl}\sigma_{ij}\sigma_{kl}\right)^{\beta} + \left(R_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn}\right)^{\gamma} + \ldots = 1$$

$$(i, j, k, \ldots = 1,2,3), \qquad (8)$$

Where R_{ij}, R_{ijkl}, R_{ijklmn} – matrix notation of tensors of the surface strength of the second, fourth, sixth and subsequent even ranks. In engineering practice, more convenient in practical applications was the next criterion is the strength of the tensor-polynomial forms:

$$R_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} + R_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} + \ldots = 1$$

$$(i, j, k, m, l, n = 1,2,3), \qquad (9)$$

Which is easily obtained from (8), taking $\alpha, \beta, \gamma, \ldots = 1$. Most of the known polynomial strength criteria are usually a special case of criterion (9). Using the strength criterion (9) in the form of

$$R_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} + R_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} = 1$$

$$(i, j, k, m, l, n = 1,2,3), \qquad (10)$$

Let us consider a destruction condition of multi-layered composite as a whole. An assumption about an independent loading, a linear-elastic behavior of material, and about an absence of interaction between the layers reduced the number of strength tensors in an equation (10), which was derived for an orthotropic composite in a plane stress state, to ten. A criterion of layered composite strength (10) turned to be very complicated for a practical application, since to find tensor coefficients of the surface strength, one needed sophisticated experiments. In most cases a destruction of multi-layered composite material started from a destruction of one layer or a break of interface bonds. Therefore, the destruction is assumed to be localized in one layer and the strength criterion should be derived namely for this layer when an ultimate surface is calculated. An approximation of the ultimate surface strength for an orthotropic layer using a quadratic polynomial is considered in [5]. The equation (10) has the following form:

$$R_{ij}\sigma_{ij} + R_{ijkl}\sigma_{ij}\sigma_{kl} = 1; \; (i, j, k, l = 1,2,3) \qquad (11)$$

Where R_{ij}, R_{ijkl} – tensors of the surface layer of the strength of the second and fourth orders. In the case of plane stress equation (11) represents the limit surface (ellipsoid) in three-dimensional stress space

$$R_{11}\sigma_{11} + R_{22}\sigma_{22} + 2R_{12}\sigma_{12} + R_{1111}\sigma_{11}^2 + R_{2222}\sigma_{22}^2 + 4R_{1212}\sigma_{12}^2 + 2R_{1122}\sigma_{11}\sigma_{22} + 4R_{1112}\sigma_{11}\sigma_{12} + 4R_{2212}\sigma_{22}\sigma_{12} = 1. \qquad (12)$$

Coefficients of equation (12) are determined using the experimentally determined limiting strength characteristics σ_{ij}^{+}, σ_{ij}^{-} $(i, j = 1,2)$. The index "+" means that this component - the ultimate tensile stress, the index "-" denotes the ultimate stress in compression. For the tensor components of the surface resistance (12) in [5] proposed the following relationship: $R_{11} = \frac{\sigma_{11}^{-} - \sigma_{11}^{+}}{\sigma_{11}^{-}\sigma_{11}^{+}}$; $R_{22} = \frac{\sigma_{22}^{-} - \sigma_{22}^{+}}{\sigma_{22}^{-}\sigma_{22}^{+}}$; $R_{12} = \frac{\sigma_{12}^{-} - \sigma_{12}^{+}}{\sigma_{12}^{-}\sigma_{12}^{+}}$;

$R_{1111} = \frac{1}{\sigma_{11}^{-}\sigma_{11}^{+}}$; $R_{2222} = \frac{1}{\sigma_{22}^{-}\sigma_{22}^{+}}$; $4R_{1212} = \frac{1}{\sigma_{12}^{-}\sigma_{12}^{+}}$;

$$2R_{1122} = \frac{R_{11} - R_{22}}{\sigma_{12}^{-}} + R_{1111} + R_{2222} - \frac{1}{\left(\sigma_{12}^{-}\right)^2} \quad . \qquad (13)$$

The strength tensor in (12) and (13) takes into account a possible difference of strength characteristics of a material tension and a compression. We should like to note that the material strength does not depend on a sign of ultimate values of strength tangents, i.e. $\sigma_{12}^{-} = \sigma_{12}^{+}$. In addition, an identity $R_{1112} = R_{2212} = 0$ is valid for an orthotropic material in symmetry axes. Existing experimental values σ_{ij}^{+}, σ_{ij}^{-} $(i, j = 1,2)$ are insufficient for a determination of components of strength tensors of R_{1122} type, therefore, to obtain and validate an empirical dependence R_{1122}, a necessity to perform exactly planned experiments arises. As a rule, the majority of methods applied to construct an ultimate interface is based on an assumption that the reinforced material is a set of anisotropic layers, which in its turn, entails a study of properties of every individual layer under loading. A theory of layered medium enables a changeover from a composite averaged stress and deformation to a local stress and deformation in any layer. We should like to note that with the exception of some individual works, all approaches do not take into account a stress and a deformation in an in-plane shear σ_{i3}^{-}, σ_{i3}^{+} $(i = 1,2)$ and a transversal break off or a compression σ_{33}^{+}, σ_{33}^{-}. An essential difference in ultimate characteristics of load-carrying layer and properties of intermediate interface layer dictates a selection of one or another model of discrete-structure theory for a plane and a shell. It becomes evident that the layering should be considered not as an isolated type of destruction but as a factor, which determines a type of discrete-structural model for a multi-layered construction. In such a way, to evaluate an effect of weakened interface contact of layers, a criterion (12) should be written in a modified form.

$$R_{11}\sigma_{11} + R_{22}\sigma_{22} + R_{33}\sigma_{33} + R_{1111}\sigma_{11}^2 + R_{2222}\sigma_{22}^2 + R_{3333}\sigma_{33}^2 + 4R_{1212}\sigma_{12}^2 + 4R_{1313}\sigma_{13}^2 + 4R_{2323}\sigma_{23}^2 + 2R_{1122}\sigma_{11}\sigma_{22} + 2R_{1133}\sigma_{11}\sigma_{33} + 2R_{2233}\sigma_{22}\sigma_{33} = 1, \qquad (14)$$

Where a tensor of the surface resistance (13) follows by analogy to add additional components:

$$R_{33}=\frac{\sigma_{33}^{-}-\sigma_{33}^{+}}{\sigma_{33}^{-}\sigma_{33}^{+}};\quad R_{3333}=\frac{1}{\sigma_{33}^{-}\sigma_{33}^{+}};\quad 4R_{1313}=\frac{1}{\sigma_{13}^{-}\sigma_{13}^{+}};$$

$$4R_{2323}=\frac{1}{\sigma_{23}^{-}\sigma_{23}^{+}};$$

$$2R_{1133}=\frac{R_{11}-R_{33}}{\sigma_{13}^{-}}+R_{1111}+R_{3333}-\frac{1}{\left(\sigma_{13}^{-}\right)^2};$$

$$2R_{2233}=\frac{R_{22}-R_{33}}{\sigma_{23}^{-}}+R_{2222}+R_{3333}-\frac{1}{\left(\sigma_{23}^{-}\right)^2}. \quad (15)$$

It is assumed that the interlayer shear strength of the material does not depend on the sign of the transverse shear stresses, i.e. $\sigma_{13}^{+}=\sigma_{13}^{-};\ \sigma_{23}^{+}=\sigma_{23}^{-}$. To use the modified criterion (14) and (15) to experimentally determine the limiting characteristics of the layer on the transverse shear and transverse compression or separation.

IV. NUMERICAL RESULTS AND DISCUSSION

A three-layer material at the level of stress σ_{11} = 115 MPa , external layers of which have $h^{(1)}=2.5\cdot10^{-3}m$ thickness, is considered. Its internal layer has – $2h^{(2)}=0.5\cdot10^{-3}м$ thickness and physical-mechanical characteristics of transversal isotropic external layers are: $E_1^{(1)}=E_2^{(1)}=1.5\cdot10^4 MPa,$

$G_{13}^{(1)}=G_{23}^{(1)}=1.715\cdot10^3 MPa,\qquad \nu_{13}^{(1)}=\nu_{23}^{(1)}=0{,}242.$ A material of adhesive average layer is regarded as isotropic: $E_1^{(2)}=E_2^{(2)}=E=3.5\cdot10^3 MPa,$

$$G=G_{13}^{(2)}=G_{23}^{(2)}=\frac{E}{2(1+\nu)}=1.296\cdot10^3 MPa,$$

$\nu_{13}^{(2)}=\nu_{22}^{(2)}=\nu=0{,}35.$ The tension strength limit of adhesive layer is considered to be equal to: $\sigma_{22}^{(2)+}=25MPa$, and the in-plane shear strength limit – to $\sigma_{13}^{(2)-}=\sigma_{13}^{(2)+}=16MPa$. in transverse shear. As it follows from the expressions in (4) - (6) there are several possible options for the appearance of new cracks (Fig. 2). Cracks, which are parallel to an initial crack, are induced by a stress $\sigma_{11}^{(2)*}$. Additional cracks, which appear at a layer interface, are a result of transverse stress $\sigma_{13}^{(2)*}$ and cracks at the interface between layers or within layers of the interlayer of the normal stresses $\sigma_{33}^{(2)*}$, as well as cracks in the second layer associated with the combination of stresses.

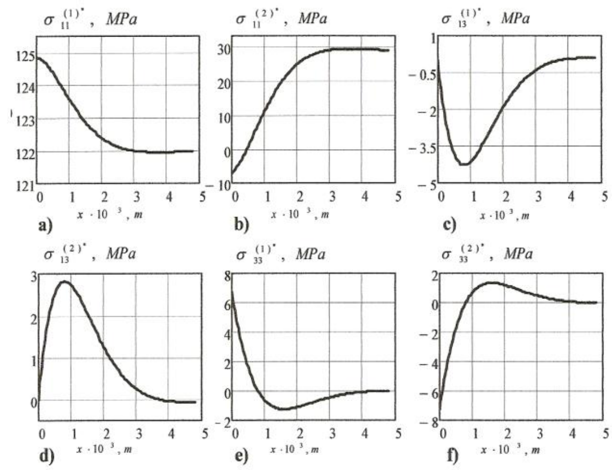

Figure 2 State of stress in the crack

Results of many experiments demonstrate that, as a rule, the first way of crack formation is more typical for the case of sample tension. In this case, a distance between cracks, according to (6), is equal to $x=\pi/k_2$. An analysis of a stressed state of a block of adjacent layers was performed using (7). Plots of stress changes $\sigma_{11}^{(2)*},\ \sigma_{13}^{(2)*},\ \sigma_{33}^{(2)*}$ over the block length are presented in Fig. 3. A maximum stress value $\sigma_{11}^{(2)*}$, which is found in the block center at x = 0 (Fig. 3b), is equal to .

$$\sigma_{11}^{(2)*}=\sigma_{11}^{(2)}[1-k_1/(k_2 sh\frac{\pi k_1}{2k_2})]k_2 sh\frac{\pi k_1}{2k_2}=18.04\ MPa$$

.

A probability of crack formation of the first type at a subsequent loading remains still high. New cracks divided a layer into blocks, a length of which was approximately equal to $0{,}5\pi/k_2$. Figure 4 shows plots of stresses $\sigma_{11}^{(2)*},\ \sigma_{13}^{(2)*},\ \sigma_{33}^{(2)*}$ for a block of $0{,}5\pi/k_2$ length. A comparison of Fig. 3 and Fig.4 indicates that an average distance between cracks decreased practically at the same $\sigma_{11}^{(2)}$ value. An average distance between cracks is $(2...2.5)h^{(2)}$. The second, third, and fourth way of new crack formation were developing simultaneously with the first one. This resulted in a formation of layering regions at an interface between the external and internal layer. An accent on two main types of damages (a matrix crack, which was located over a layer depth $h^{(2)}$, in parallel to a transverse fiber and a layering), however, should not distract from an understanding that a finite reason of destruction of a layered composite material is a fiber break.

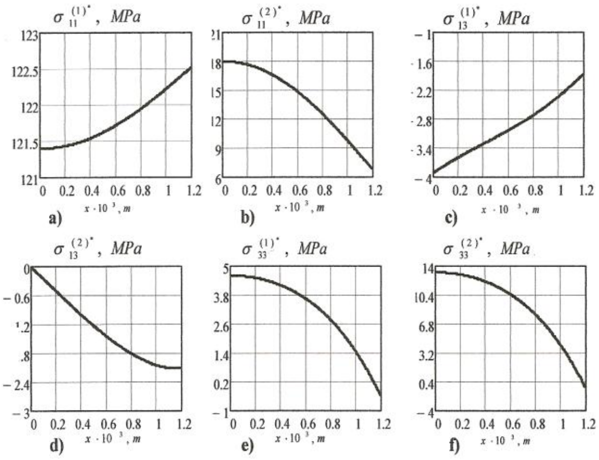

Figure 3 Stress state of a single block of the middle layer of length π / k_2

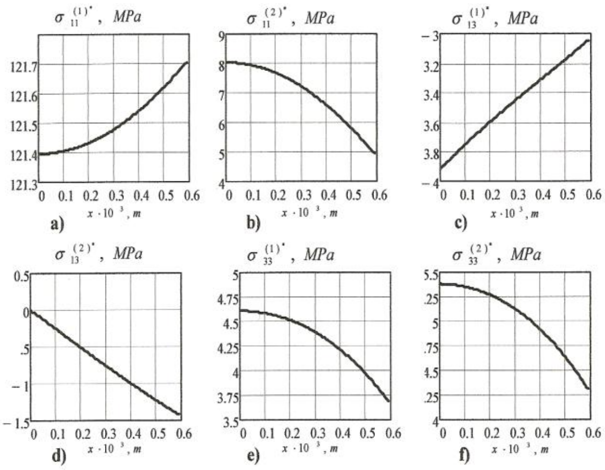

Figure 4 Stress state of a single block of the middle layer of length $0.5\pi / k_2$

A stressed state of cylindrical sample made of glass reinforced plastic of 0.1m to 0.2m length, 0.09m diameter, and 0.002m thickness was studied. The cylinder was made of four glass fabric layers TG 430 – C (100). A polyester orthophthalic resin Cristic 2 – 446 PA with a reduced sterol emission was used as a binder. Table 1 and Table 2 present physical-mechanical characteristics of studied glass-reinforced plastic samples [6].

TABLE 1 Experimental and theoretical values of elastic characteristics of the GRP.

E_{ij}, *MPa*	**G_{ij}, *MPa***	ν_{ij}	ν_{ji}
$E_{11} = 15000$ $E_{22} = 15000$ $E_{33} = 7689$	$G_{12} = 2554$ $G_{13} = 2187$ $G_{23} = 2187$	$\nu_{12} = 0{,}12$ $\nu_{13} = 0{,}41$ $\nu_{23} = 0{,}41$	$\nu_{21} = 0{,}12$ $\nu_{31} = 0{,}21$ $\nu_{32} = 0{,}21$

TABLE 2 Experimental values of limiting stresses GRP

$\sigma_{11}^{+} = \sigma_{22}^{+}$, *MPa*	$\pm a_{\sigma_{cp}}$, *MPa*	$\sigma_{11}^{-} = \sigma_{22}^{-}$, *MPa*	$\pm a_{\sigma_{cp}}$, *MPa*	$\sigma_{11}^{u} = \sigma_{22}^{u}$, *MPa*	$\pm a_{\sigma_{cp}}$, *MPa*
200	7	180	8	160	4

Table 2 presents a confidence interval of ultimate stress average value $\pm a_{\sigma_{cp}}$ for a confidence probability $1 - \alpha = 0.95$. We should like to note that a spread of experimental values obtained for and ultimate destructive stress of an in-plane shear and compression is very high, which, first of all, is due to a structure feature of reinforced plastics, laborious and complicated conditions of experiment realization. Therefore, to perform further researches, we have to accept average values $\sigma_{33}^{-} = 90 MPa$, $\sigma_{33}^{+} = 16 MPa$, $\sigma_{13}^{-} = \sigma_{13}^{+} = \sigma_{23}^{-} = \sigma_{23}^{+} = 30 MPa$, $\sigma_{12}^{-} = \sigma_{12}^{+} = 50 MPa$, which are based on experimental data presented in a work [7] for a glass reinforced plastic of similar structure. The internal shells were loaded by an air using a special device [8]. Theoretical and experimental results were obtained for a cylinder with stationary coupled ends. Values of normal stresses σ_z, σ_θ in a transversal and circumference direction, respectively, as well as a stress of in-plane shear σ_{rz} under an action of internal hydrostatic pressure of q intensity were specified. To evaluate a load-carrying ability of the considered glass-reinforced plastic shell, a modified criterion of strength (14) and the following values of material ultimate strength $\sigma_z^{+} = \sigma_\theta^{+} = 200 MPa$, $\sigma_z^{-} = \sigma_\theta^{-} = 180 MPa$, $\sigma_z^{-} = 90 MPa$, $\sigma_z^{+} = 16 MPa$, $\sigma_{\theta z}^{-} = \sigma_{\theta z}^{+} = 50 MPa$, and $\sigma_{rz}^{-} = \sigma_{rz}^{+} = \sigma_{\theta z}^{-} = \sigma_{\theta z}^{+} = 30 MPa$. should be employed. We should like to note also that a special attention should be paid to a momentum-free plane stressed state. In this case, a value of circumferential stress reported in [8], which were obtained using an improved theory of multi-layered shell discrete structure coincided fairly well with a value of circumferential stresses obtained by a formula $\sigma_\theta = qr/h$. A main difference of results is found in a region of stationary coupled shell ends. A significant tangential stress σ_{rz} was found at a distance of shell thickness from its end, as it was reported in [8], which in a combination with a normal stress σ_z resulted in a destruction of this shell.

A theoretical value of damaging pressure intensity is based on an offered modified multi-nominal strength criterion (14). Theoretical values of corresponding stresses [8] were correlated with the help of analytical dependences (3) – (7). An intensity of theoretically found destructing pressure was

$q^* = 2.6 MPa$. The destruction occurred in a zone of cylinder stationary coupled ends, and this value was a little lower than an experimentally obtained destructing pressure $q_Э^* = 2.65 MPa$.

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