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Comparison of the Dynamic Behavior of a Manipulator with Two Flexible Arms

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Abstract-This work is devoted to study the modal analysis of manipulator with flexible arms by the finite element method, based on beam theory. The finite element method is used to calculate the matrix and global stiffness of the structure, then the eigenfrequencies. The element chosen is a bending beam with three degrees of freedom. Modal analysis of the manipulator is based on the analytical method used to solve the equation of free vibrations. To validate the effectiveness of our program sets, two models are studied: the first with nine nodes and the second with twenty-one nodes. The programming of this resolution method was performed under the Matlab environment. In this study, a comparison is made between the rigid and flexible structure. This study showed that the position of the end of the robot arm in the flexible case is sensitive to the first frequency; hence the control in this range of low eigenfrequencies is needed.

Keywords*- modal analysis; modeling; finite element method; rigid manipulator; flexible manipulator.*

I. INTRODUCTION

Robotic manipulators are widely used to help in dangerous, monotonous, and tedious jobs. Most of the existing robotic manipulators are designed and built in a manner to maximize stiffness in an attempt to minimize the vibration of the end-effector to achieve a good position accuracy. This high stiffness is achieved by using heavy material and a bulky design. Hence, the existing heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating payload [1-2]. It will be necessary to calculate the natural frequencies of a structure, which is very essential to improve its behavior during operation. In this perspective, an analytical method of resolution is developed based on the equation of free vibrations to determine the eigenfrequencies of a robot manipulator with two flexible arms and the effect of flexibility on the end of arm of robot.

II. FINITE ELEMENT METHOD

The finite element method (FEM) is one of the most widely technique for solving partial differential equations by first discretising these equations in their space dimensions [3-4]. To resolve the natural frequencies and mode shapes of structure, it will be necessary to calculate the global matrix of mass and stiffness [6]. The calculation of mass and stiffness matrix of each beam element is first on the local coordinate system as shown in figure1. In this study, we limit the number of degrees at three degrees of freedom per node (two displacements: U and V and a rotation (θ) around Z axis).

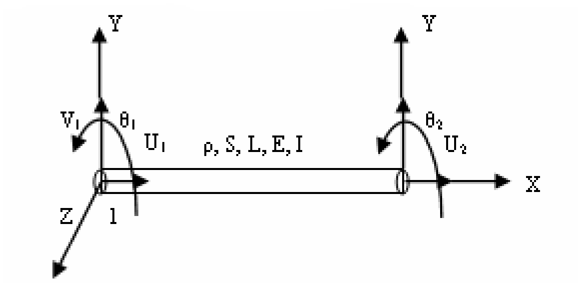

Figure 1. Beam element with six degrees of freedom.

If we put:

$$a = \frac{EI}{L^3} \qquad b = \frac{ES}{L} \tag{1}$$

Where:

E: the Young modulus.

I: the moment of inertia of the beam.

L: the length of the beam.

S: the section of the beam.

ρ: the density.

$$K_e = \begin{bmatrix} b & 0 & 0 & -b & 0 & 0 \\ 0 & 12a & 6aL & 0 & -12a & 6aL \\ 0 & 6La & 4aL^2 & 0 & -6aL & 2aL^2 \\ -b & 0 & 0 & b & 0 & 0 \\ 0 & -12a & -6aL & 0 & 12a & -6aL \\ 0 & 6L & 2aL^2 & 0 & -6aL & 4aL^2 \end{bmatrix} \quad (2)$$

$$M_e = \frac{\rho SL}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix} \quad (3)$$

The global matrix of mass and stiffness are obtained by using assembly method:

$$K_G = B^T K_{des} B$$
$$M_G = B^T M_{des} B \quad (4)$$

Where: B is the Boolean matrix; K_{des}, M_{des} are unassembled matrix, they contain only elementary matrix of mass and stiffness.

$$K_{des} = \begin{bmatrix} \left[K_e{}^1\right] & 0 \\ 0 & \ddots \left[K_e{}^N\right] \end{bmatrix} \quad (5)$$

$$M_{des} = \begin{bmatrix} \left[M_e{}^1\right] & 0 \\ 0 & \ddots \left[M_e{}^N\right] \end{bmatrix} \quad (6)$$

Figure 2 shows a system with two flexible arms in initial configuration and in deformed state. The modeling of each arm is based on Timoshenko beam theory. In this case, we indicate the fixed system of original coordinates OXY. The angle on each arm is θ_i. The deflection and angle of rotation of the normal with the neutral axis are represented in the local coordinates by w_i and ψ_i respectively. Vector r_{oi} indicate the position of any point on the arm i :

$$r_{oi} = r_i + T_i \ R_i \quad (7)$$

The vector r_i represents the relative position to the origin of coordinates of the arm i and R_i is the elastic deformation of the arm i in the local coordinate system. T_i is the transformation matrix that connects the local coordinate system fixed to the system:

$$R_i = \begin{Bmatrix} x_i + u_i \\ y_i + w_i \end{Bmatrix} \quad (8)$$

$$T_i = \begin{bmatrix} \cos(\beta_i) & \sin(\beta_i) & 0 \\ -\sin(\beta_i) & \cos(\beta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

In these equations, u_i is the local displacement in the direction x_i and w_i is the displacement in the direction y_i and β_i is the rotation of joint i with respect to the origin of the fixed coordinate system:

$$\beta_i = \begin{cases} \theta_i \rightarrow i=1 \\ \sum_{j=1}^{i} \theta_j + \sum_{j=1}^{i-1} \left(\psi_j\right)_{1j} \rightarrow i=2,3,... \end{cases} \quad (10)$$

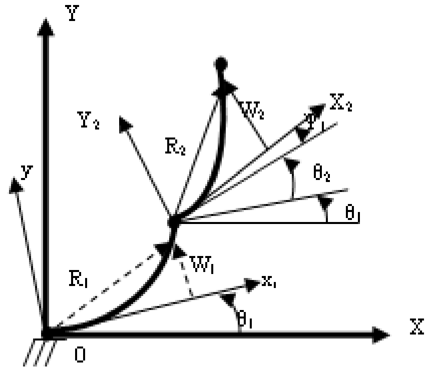

Figure 2. Manipulator modeling with two flexible arms.

III. RESOLUTION OF THE EIGENVALUS PROBLEM

The equation of motion (undamped and free vibration):

$$m\ \ddot{q}(t) + k\ q(t) = 0 \quad (11)$$

The equation (11) can be written in matrix form:

$$\left[M\right]\left\{\ddot{q}\right\} + \left[K\right]\left\{q\right\} = 0 \quad (12)$$

With q: the vector of degrees of freedom. The general solution of equation (11) is:

$$\{q\} = \{q_0\} e^{i\omega t} \quad (13)$$

By substituting the equation (13) in to equation (12), ω is a solution of eigenvalue problem as follows [5-7]:

$$\left[K\right]\{q_0\} = \omega^2 \left[M\right]\{q_0\} \quad (14)$$

Then, the determinant must be zero:

$$det\left(\left[K\right] - \omega^2 \left[M\right]\right) = 0 \quad (15)$$

There are many methods to calculate the eigenvalues; the most of these methods are to write the equation (14) as follows:

$$\left[H\right]\{X\} = \lambda \{X\} \quad (16)$$

Where [H] is a positive and symmetric matrix, it is clear that if we write directly the equation (14) as:

$$[K]^{-1}[M]\{q_0\} = \frac{1}{\omega^2}\{q_0\} \quad (17)$$

Where $[K]^{-1}$ is the inverse of the matrix [K], the symmetry property is not always preserved. Therefore, it is necessary to write the matrix [K] using the Cholesky decomposition:

$$[K] = [L]^T [L] \quad (18)$$

$[L]^T$is the transpose of the matrix [L] and [L] is a lower triangular matrix. The equation (14) is written:

$$[L]^{-T}[M][L]^{-1}[L]\{q_0\} = \frac{1}{\omega^2}[L]\{q_0\} \quad (19)$$

By writing equation (19) as similar form as equation (14):

$$[H] = [L]^{-T}[M][L]^{-1} \quad (20)$$

$$\{X\} = [L]\{q_0\} \quad (21)$$

$$\lambda = \frac{1}{\omega^2} \quad (22)$$

Notes:

- [K] and [M] are positive.
- If the matrix [K] is singular, it has not inverse if we use the general methods above, it will be necessary to introduce a parameter of the same magnitude as ω^2 to solve the following problem:

$$([K] + \alpha[M])\{q_0\} = \left(\omega^2 + \alpha\right)[M]\{q_0\} \quad (24)$$

The new matrix [K]+α[M] has an inverse, and the solution is $\omega^2 + \alpha$.

IV. Example of Application

Table I shows the physical and mechanical properties of a manipulator with two flexible arms taken in this example.

TABLE I. PHYSICAL AND MECHANICAL PROPERTIES OF A MANIPULATOR WITH TWO FLEXIBLE ARMS

Property	Value
Elasticity modulus	E = 71 GPa
Material density	$\rho = 2710$ kg/m^3
Section	S = 0.0012 m^2
Arm length	L = 1m
Moment of inertia	I = 9×10-8 m^4
θ_1	45°
θ_2	-15°

V. Results

To validate the effectiveness of our program sets, two models are calculated, the first contains nine nodes and the second contains twenty-one nodes as observed in table II.

TABLE II. RESULTS OBTAINED BY THE PROGRAM AND ABAQUS SOFTWARE

Mode	Eigenfrequencies (rd/s)		
	9 nodes (8 elements)	*21 nodes (20 elements)*	*Abaqus Software*
1	15.49	15.15	14.03
2	15.48	7.49	7.74
3	15.47	7.07	7.10

A. Mode Shapes for Nine Nodes With three Degrees of Freedom

The deformed shapes are shown in the figures 3 – 5 for the first three modes for nine nodes with three degrees of freedom.

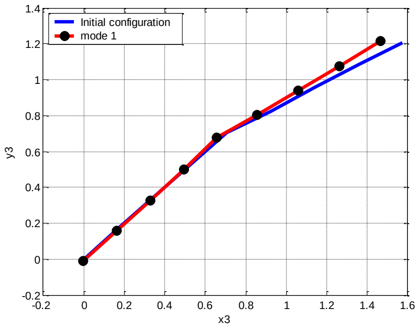

Figure 3. Mode 1.

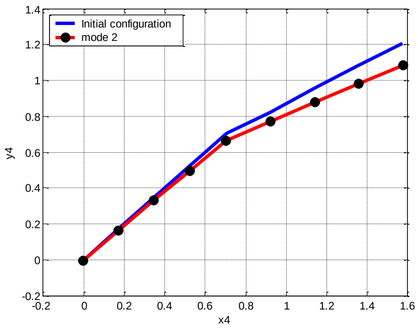

Figure 4. Mode 2.

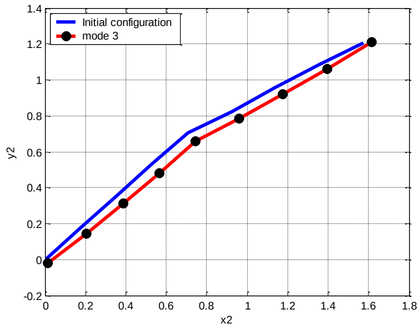

Figure 5. Mode 3.

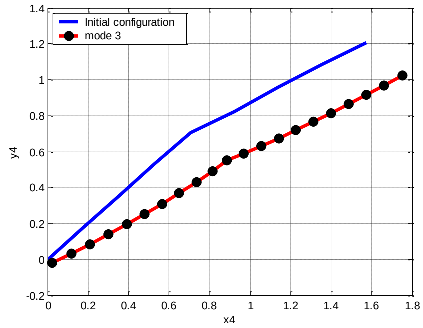

Figure 8. Mode 3.

B. Mode Shapes For Twenty nodes With three Degrees of Freedom

The deformed shapes are shown in the figures 6 – 8 for the first three modes for twenty nodes with three degrees of freedom.

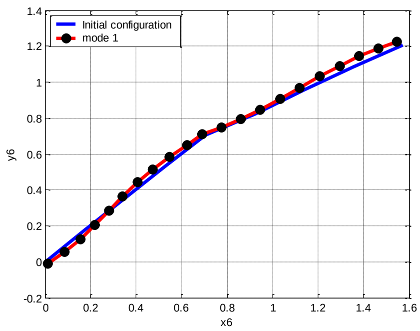

Figure 6. Mode 1.

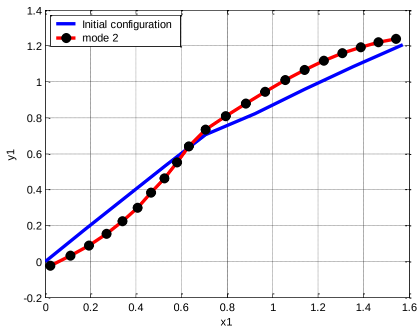

Figure 7. Mode 2.

VI. COMPARISON BETWEEN THE RIGID STRUCTURE AND FLEXIBLE STRUCTURE

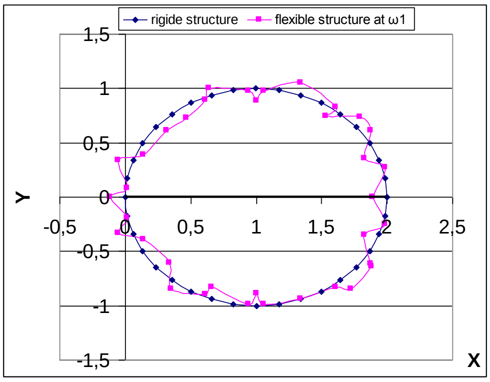

Figure 9. The first Angular frequency ω1.

This comparison is done for the end of second arm in the case of displacement between the rigid structure and flexible structure [8] in the case when $\theta_2 = 0°$ and $\theta_1 = 0°$ to 360° as shown in figure 9 where the control of end arm of robot is needed.

VII. CONCLUSION

The established method was used to determine the natural frequencies and mode shapes of a flexible manipulator with two arms. The modeling is done by the finite element method based on beam theory. The technical calculation tool (Matlab) helped us to determine the matrix mass and stiffness matrix of the structure, the eigenvalues, and eigenvectors of structure and mode shapes of the structure in each mode. To validate the effectiveness of our program, two models are calculated, the first at nine nodes and the second twenty –one nodes with three degrees of freedom per node. Modal results obtained by the model are compared with those calculated by Abaqus software.

This study showed that the position of end of the robot arm in the flexible case is sensitive to the first frequency; hence the control in this range of low eigenfrequencies is needed.

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